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Measurement of Material Thickness using X-Ray Attenuation

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By

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The 60 kev x-rays from Americium-241 (^{241}Am) have been used in an x-ray attenuation experiment to measure the thickness and attenuation coefficient of an aluminum alloy. Using traditional measurement tools such as a micrometer, to determine the thickness and uniformity of soft metal targets and curved aluminum target cell windows is challenging. Furthermore, the determination of window thickness is important to Jefferson Lab experiments, in particular the Qweak experiment.

In this thesis, the thickness of Aluminum foil AL7057 is determined with high accuracy using x-ray attenuation. Using the x-ray attenuation technique has the advantage of non-destructive measurement.

Key words: thesis, photon interactions, measurement, x-rays, attenuation

DEDICATION

To my parents

ACKNOWLEDGEMENTS

To my life-coach, my mother : because I owe it all to you. Many Thanks!

My eternal cheerleader, Aisha : I miss our interesting and long-lasting chats. My forever interested, encouraging and always enthusiastic my wife : she was always keen to know what I was doing and how I was proceeding.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS, ABBREVIATIONS, AND NOMENCLATURE

^{241}**Am** An isotope of the element Americium with an atomic weight of 241 amu

amu atomic mass units

Al Aluminum

Cr Chromium

Cu Copper

eV electron volt - the amount of energy required to move an electron in a potential difference of one volt

Fe Iron

Ge Germanium

ICET Institute of Clean Energy Technology

JLab Thomas Jefferson National Accelerator Facility (Jefferson Lab)

Mg Magnesium

Mn Manganese

Ni Nickel

NIST National Institute of Standards and Technology

^{237}**Np** An isotope of the element Neptunium with an atomic weight of 237 amu

Research Park Thad Chocran Research, Technology, and Economic Development Park

Si Silicon

Ti Tin

Zn Zinc

CHAPTER 1

INTRODUCTION

1.1 Motivation

The recently completed Qweak experiment at Jefferson Laboratory made the first direct determination of the proton's weak charge, Q_W^p via a measurement of the parity-violating asymmetry in elastic electron-proton scattering at low four-momentum transfer [11].

A liquid hydrogen target used in the Qweak experiment was contained in an aluminum cell with very thin aluminum windows of the order of 3-4 thousands of an inch. The thickness of the aluminum windows needed to be accurately measured. Due to the pressure in the target, the thin window was curved in shape. Since the window is curved, measurements using a micrometer would require a spherical face or point micrometers. These micrometers have precisions of 0.1 mil, which is not accurate enough for our 3-4 mil thick aluminum windows. There are sub-micro micrometers with accuracies of 0.02 mil; however, the advantage of x-ray attenuation that it allows minimal physical interface with the window compared to the traditional method and it better matches the material.

A previous study was performed at MSU using a similar setup, but lacked the ability to remotely move the target foil[4]. We have enhanced the setup to include an x-y two axis stage to allow remote positioning of the material being studied. In addition, this enhanced setup will allow 2-D scanning of a target face.

1.2 Experiment Overview

In this experiment, we measure the thickness of a thin aluminum window using x-ray attenuation. The process consists of sending a beam of x-rays through the foil. The motion control was used to precisely move the target material into the container. The thickness of the foil can be determined from the change in the intensity of x-rays that pass through the aluminum window in comparison to the initial intensity of the x-rays source. The change in intensity is due to photon interaction with the material, which attenuates the beam. This attenuation is governed by the thickness and attenuation coefficient of the material, which can be measured for the same material using another foil which is flat not curved with a known thickness.

CHAPTER 2

THEORY

2.1 Radiation

Radiation is the release of energy in the form of moving particle waves. This energy can be low such as microwaves from a cell phone or high such as x-rays or cosmic rays from outer space. The streams of particles can be ionizing or non-ionizing radiation depending on their energies. The nucleus of an atom is made up of two particles; protons, which carry a positive charge, and neutrons which have no charge. Outside the nucleus are electrons, which carry a negative charge. The Coulomb attraction of these negative electrons to the positive nucleus is what keeps the atom together. In fact, every element has a specific number of protons and neutrons. However, when atoms have too many or few neutrons, the atom becomes unstable or "radioactive". The radioactive decay or nuclear decay is the process by which the nucleus of an unstable atom loses energy by emitting radiation, including alpha particles, beta particles and gamma rays. The α-decay involves the emission of helium nuclei while β-decay involves the emission of electrons or anti-electrons, called a positrons, and γ-decay involves the emission of photons. The radioactivity involved with this attenuation experiment is γ-decay in the form of x-rays energy photons [10]. However, these x-rays come from the exited daughter nucleus, resulting from α decay of our source ^{241}Am.

2.2 Photon Interactions

Photons are electromagnetic radiation with no mass and charge. As photons travel through a matter, they can undergoing interactions which will lead to partial or total transfer of the photon's energy. For example, the possible interactions with an electron in an atom are Rayleigh scattering, photoelectric effect, Compton scattering, and pair production.

Rayleigh scattering is named after the British physicist Lord Rayleigh [9]. During Rayleigh scattering, the electric field of the incident photon, causes all the electrons in the scattering atom to oscillate in phase. The atom's electrons cloud immediately radiates energy, emitting a photon of the same energy [2]. Rayleigh scattering refers to the scattering of light or electromagnetic wave by particles which are smaller than the photon wavelength

Compton scattering is the interaction of photons with electrons, where the electrons lose energy to the photon, i.e, the wavelength of the photon decreases. The photon scatters off the electron according to the relation:

$$\cot(\phi) = (1+\gamma)\tan\frac{\theta}{2} \quad \text{with } \gamma = \frac{h\nu}{m_e c^2} \tag{2.1}$$

Where h is Plank's constant, ν is the frequency of the photon, m_e is the mass of an electron, c is the speed of light, θ is the photon's scattering angle and ϕ is the recoiling electrons scattering angle.

Pair production is a process that converts a photon's energy into an electron-positron pair. However, this process only occurs for photon energies above 1.022 MeV [**?**].

The photoelectric effect is a phenomenon where the photon striking a metal surface causes electrons to be ejected from the metal. The energy of the incident photon must be greater than or equal to the binding energy or work function of electron. This condition is given by

$$E_i = E_e + E_b \tag{2.2}$$

where E_i is the energy of the incident photon, E_e is the energy of the ejected electron and E_b is binding energy of electron. Figure 2.1 illustrates these photon interactions with aluminum.

2.3 x-ray Production and Detection

On November $8, 1895$ Roentgen discovered x-rays for which he won the Noble Prize [1]. At that time, he used the latter "X" to refer to an unknown source. Later, Roentgen used his x-rays and a photographic plate to take a picture of his wife's hand, which displayed its skeletal structure and her wedding ring [7]. x-rays are produced by transforming the kinetic energy of the electron to electromagnetic radiation. x-rays can be generated as a

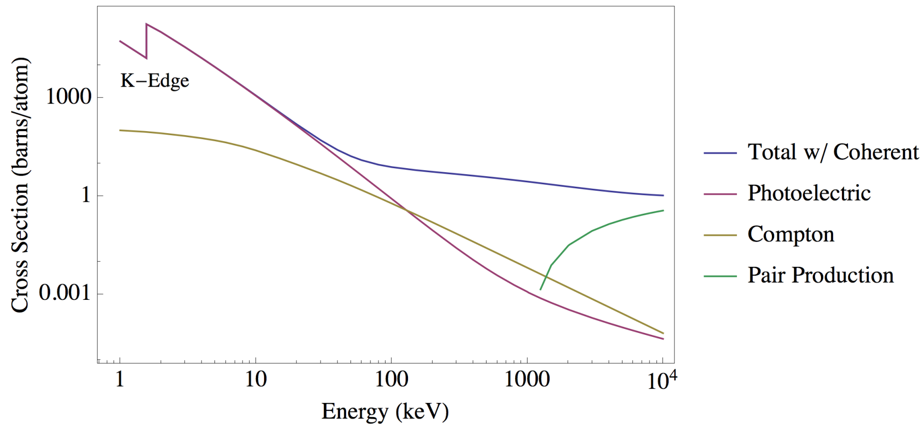


Figure 2.1

Cross section of Aluminum as a function of energy [4]

continuous spectrum called "bremsstrahlung radiation" or discrete x-rayss energy peaks called "characteristic radiation".

Electrons in the atom are distributed in shells with a specific binding energy known as K,L,M and N shells. The energy of these shells are in order from high energy to low energy binding. When the target electron is ejected from the inner shell of the atom, the atom leaves behind an unoccupied energy level. Electrons from outer-shell jump to the inner shell in such way that will emit photons with an energy level equivalent to the energy difference between the higher and lower states. "Each element has a unique set of energy levels, and thus the transition from higher to lower energy levels produces x-rays with frequencies that are characteristic to each element" [6]. "When an electron falls from the L shell to the K shell, the x-rays emitted is called a K-alpha x-rays. Similarly, when an electron falls from the M shell to the K shell, the x-rays emitted are called K-beta x-rays" [6].

Americium-241 (^{241}Am) decays into Neptunium-237 (^{237}Np) through alpha decay. Figure 2.2 shows the decay scheme for ^{241}Am. Figure 2.3 shows a log plot of the source data along with the peaks obtained from Origin software. In addition, Figure 2.4 is zoomed into low energy peak.

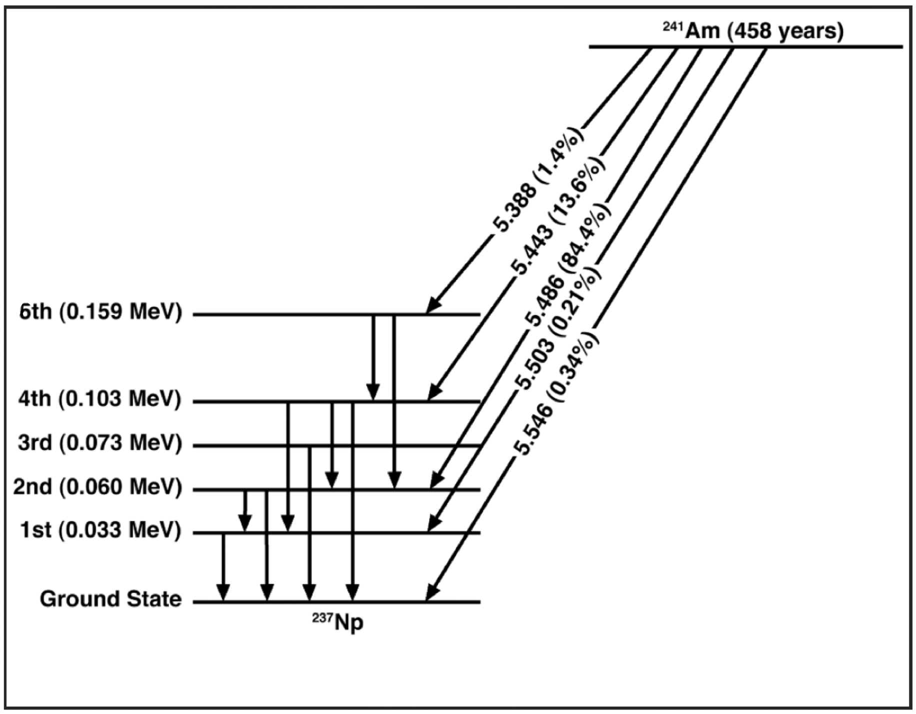
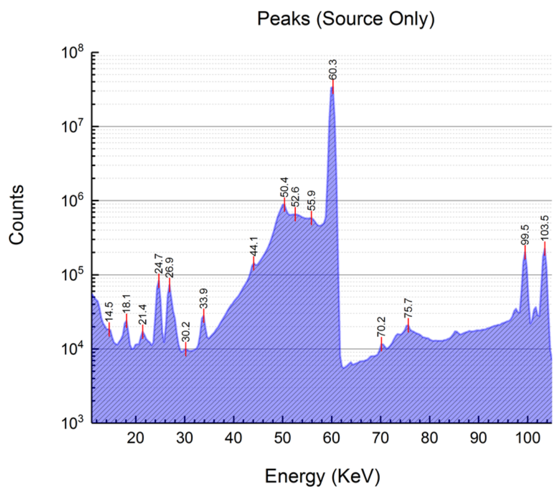


Figure 2.2

Decay Scheme of ^{241}Am with energy in MeV [8]

In the bremsstrahlung process, a high speed electron traveling in a material will decelerate and be deflected due to the forces of the atom, and will emit energy known as bremsstrahlung x-rays.



Energy (KeV)	Reaction
14.5	L_α (Np)
18.1	$L_{\beta1}$ (Np)
21.4	$L_{\gamma1}$ (Np)
24.7	unknown
26.9	$\gamma_{2,1}$ (Np)
30.2	$\gamma_{4,3}$ (Np)
33.9	$\gamma_{1,0}$ (Np)
44.1	$\gamma_{4,2}$ (Np)
52.6	Ge escape peak?
55.9	$\gamma_{6,4}$ (Np)
60.3	$\gamma_{2,0}$ (Np)
70.2	$\gamma_{4,1}$ (Np)
75.7	$\gamma_{3,0}$ (Np)
99.5	$\gamma_{2,0}$ (Np)
103.5	$\gamma_{4,0}$ (Np)

Figure 2.3

^{241}Am Peaks

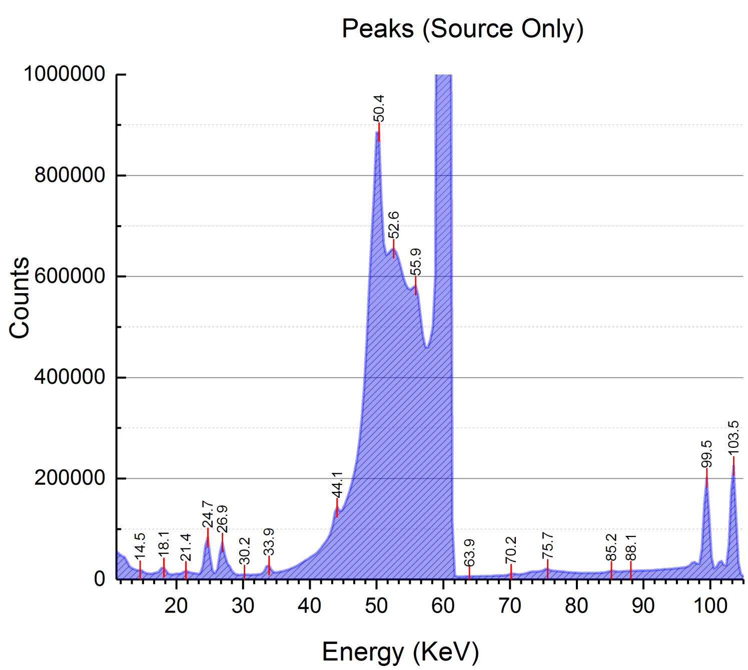


Figure 2.4

Zoomed Into Low Energy Range

2.4 Attenuation

Attenuation is the process of removing photons through absorption or scattering of an x-ray or γ-ray.

2.4.1 Linear attenuation coefficient

The attenuation coefficient is related to ability of a material to remove photons from the beam by one of the interactions described earlier. A large attenuation coefficient means that the beam is quickly attenuated as it passes through the medium, and a small attenuation coefficient means that the medium is relatively transparent to the beam. The number of removed photons is expressed as :

$$n = \mu N \triangle x \tag{2.3}$$

Where, μ is the attenuation coefficient, $\triangle x$ is the small thickness, n is the number of removed photons, and N is the number of incident photons on the material. For a mono-energetic beam of photons

$$N = N_0 e^{-\frac{\mu}{\rho}x} \tag{2.4}$$

where N_0 is the number of incident photons and N is the number of transmitted photons through a thickness x.

In other word,

$$I = I_0 e^{-\frac{\mu}{\rho}x} \tag{2.5}$$

Where I is the intensity of incident photons that passes through the material, I_0 is the initial intensity of the radiation, ρ is the density of the material, and x is the thickness of the material. The linear attenuation coefficient is directly proportional to the density of the material that the photons pass through. Also, it is the sum of individual linear attenuation coefficients for each type of interaction.

2.4.2 Mass attenuation coefficient

The mass attenuation coefficient is defined as the linear attenuation coefficient divided by the density of the material. The mass attenuation coefficient depends on the energy of the photon. The attenuation coefficient is related to the electron density which is given by

$$\frac{\mu}{\rho} = \frac{\sigma_{\text{total}}}{uA} \tag{2.6}$$

Where u is the atomic mass unit, and A is the relative atomic mass of the material. σ_{total} is the sum of the cross sections for the different photon interactions. Here,

$$\sigma_{\text{total}} = \sigma_{\text{phel}} + \sigma_{\text{ray}} + \sigma_{\text{com}} + \sigma_{\text{pair}} + \sigma_{\text{photon sc}} \tag{2.7}$$

Where σ_{phel} is the cross section due to the photoelectric effect, σ_{ray} is the cross section due to Rayleigh scattering, σ_{com} is the cross section due to Compton scattering, σ_{pair} is the cross section due to the pair production and $\sigma_{\text{photon sc}}$ is the cross section due to the photo-nuclear effect. Figure 2.5 shows the mass attenuation coefficient from NIST for Aluminum.

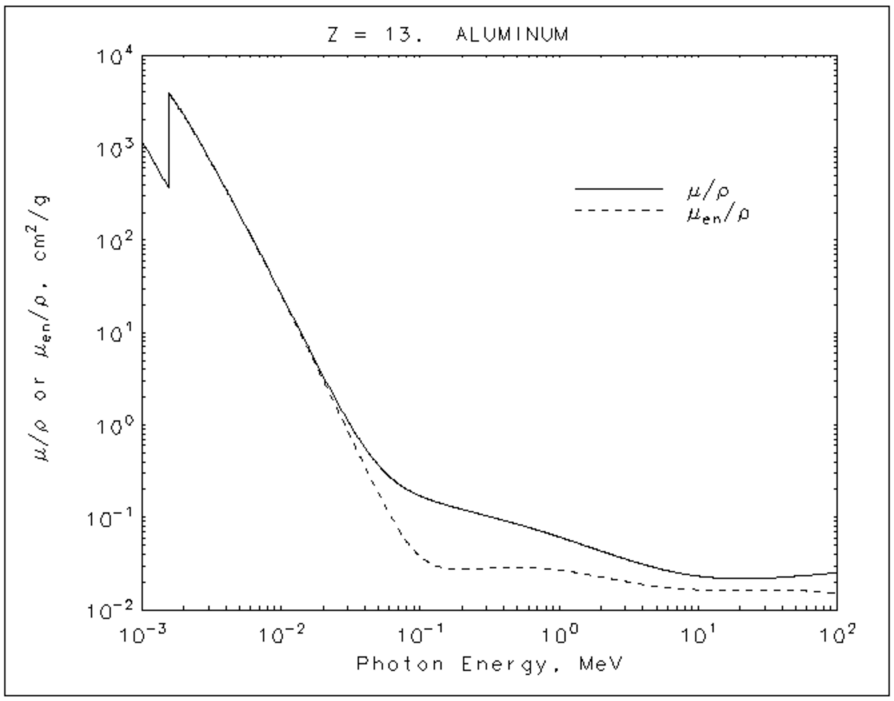


Figure 2.5

Mass Attenuation Coefficient for Aluminum [5]

CHAPTER 3

EXPERIMENT

3.1 Apparatus

In this experiment, we measured the thickness of a thin foil using an x-ray attenuation technique. This method involved using a radioactive ^{241}Am source to produce the low-energy γ rays, which were attenuated through the target material. Three measurements are required to determine the thickness of the thin curved window. First, a measurement of x-ray spectrum without any material between the ^{241}Am source and the detector. Second, a measurement with a thick ($\approx$1mm) flat target foil of the same alloy as the thin window. Third, a measurement with a thin foil. Next, the thickness of target material is determined. The radioactive source and the apparatus are located at ICET in a radiologically controlled room.

A former graduated MSU student designed the apparatus, however the design was enhanced by adding an x-y stage to remotely move the targets. The design of the apparatus was modeled using AutoCAD Inventor 2016 with components made in the ICET mechine shop. The apparatus consists of five main components: the base, the collimator, the sample holder, the source holder, and the stand. The base has a cylindrical collimator to fit over the detector and a square part on top to hold the rest of the apparatus. The base was designed to fit on top of a germanium (Ge) detector inside the shielding housing as show in Figure 3.1

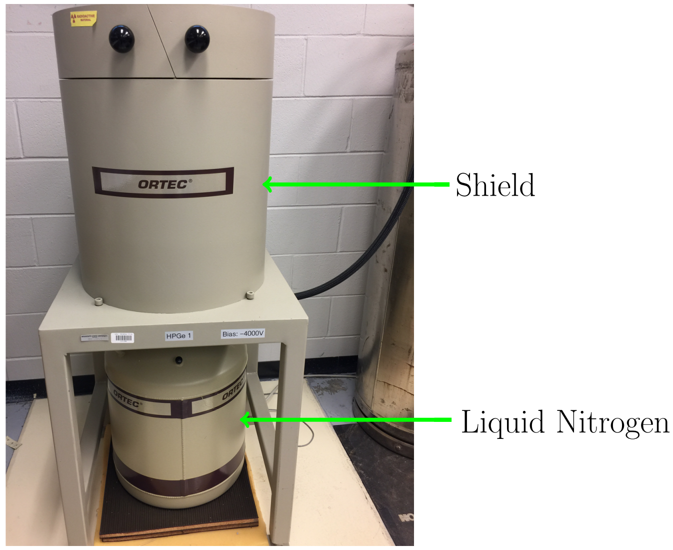


Figure 3.1

Germanium Detector Housing

The collimator is made from thick steel with density of 8.0 g/cm^3, and a thickness of 0.5 inch designed to block almost 99.997% of incidents photons. There is a hole in the center of the collimator with a diameter of half inch to allow x-rays from the source to interact with the detector. The collimator covered the detector in such a way that all unwanted photons could be stopped from reaching the detector as show in Figure 3.2 and Figure 3.3 .

The collimator has a small hole to allow a small beam of photons to pass through. The sample holder consists of two main components: the stage and the motion control manager. The plate attached to the stage was designed using AutoCAD inventor 2016. Next, the plate was sent to the mechanics shop at ICET for manufacturing. The stage has two components: the x-y stage, and the window holder. The x-y stage has four small holes

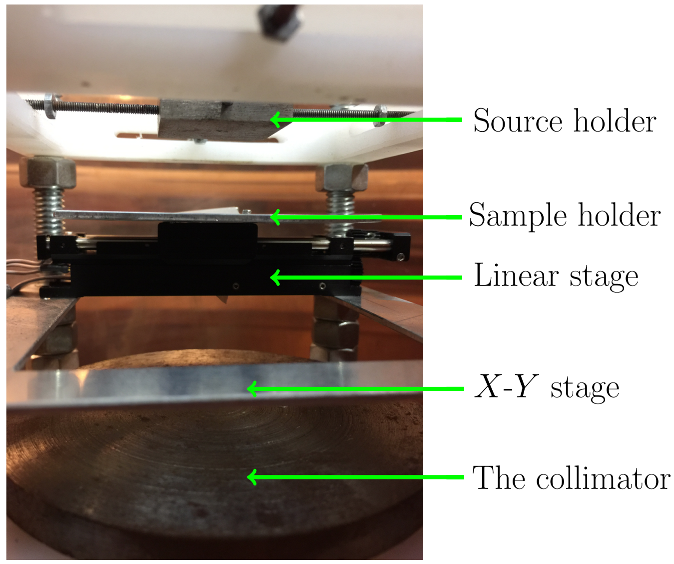


Figure 3.2

Side View of The Apparatus

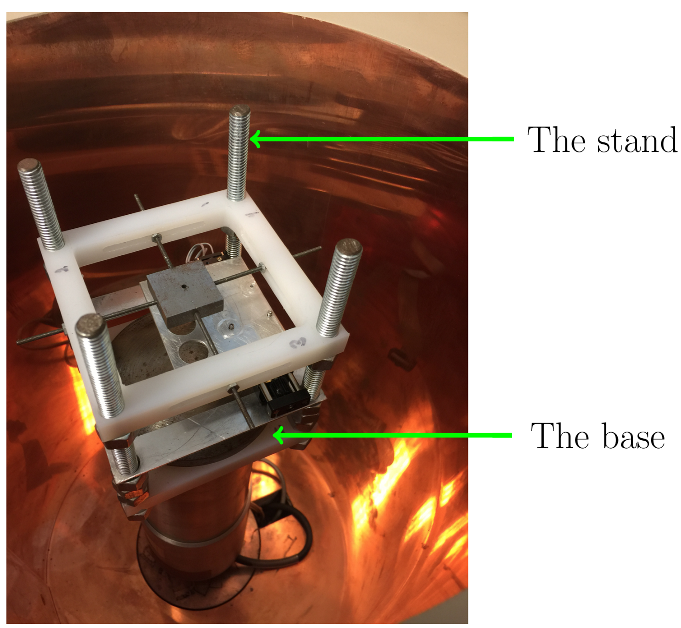


Figure 3.3

Apparatus Inside The Shield

to hold the movable axis MM-3M-F. The National Aperture motion control manger MC-CQ-4X was used to precisely control the position of the x-y stage as shows in Figure 3.4.

Figure 3.4

Motion Control Manger

The window holder is in the shape of a rectangular plate which had three holes of half an inch diameter. The target window fits into the top of circle. The source holder was designed to send a beam of photons which hit the target window and pass through the small hole of the collimator on its way to the detector. The source holder and the collimator were made from same material. The ^{241}Am was used as the γray source.

The stand was attached to four long threaded rods with two square bases that hold the sample holder and source holder. Each square base is attached to the four threaded rods through holes in each corner of the base. Steel nuts were used to secure each square base at some fixed height along the stand.

3.2 Data Collection

The data were collected at ICET by Ron Unz over a period of two weeks. The target materials were a thin Al window and a thick Al foil. The window and foil were made of the same batch of aluminum alloy, AL7075-T6. The procedure for calculating the thickness of the thin foil is by determine the mass attenuation coefficient using the measurement thick foil using the following procedure. First, the intensity of the source was measured without any material between the source and collimator. Second, the intensity of the source was measured when the thick foil was placed between the source and the collimator. Third, the intensity of the source was measured when the thin foil was placed between the source and the collimator. The explanation for data measurement process is as follows. Each data file contains the intensity result from different trials and different materials. The motion control manger is used to position the sample holder to the center of collimator. Next, place the source inside the source holder. Then, place the empty sample holder in the apparatus. The apparatus is placed inside the detector.

The germanium detector would run multiple trials for one hour and twenty-four hours and repeat the measurement for : source only, the thick foil and the thin foil. Data collection took several weeks since these measurements had to be fit in with other measurement from ICET. A simple output data froma trial is shown in Table 3.1. Figure 3.5 and Figure 3.6, below show graphs of imported data for source only.

Table 3.1

Sample data file

```
$SPEC_ID:
Molnar Range
$SPEC_REM:
DET# 1
DETDESC# HPGe Detector 1
AP# GammaVision Version 6.07
$DATE_MEA:
05/10/2017 18:05:09
$MEAS_TIM:
live time 3600, run time 3733
$DATA:
0 8191
 0
 0
 ⋮
 10764
 12961
 12961
$ROI:
0
$PRESETS:
Live Time
3600
0
$ENER_FIT:
0.137869 0.366631
$MCA_CAL:
3
1.378689E-001 3.666307E-001 -4.415950E-008 keV
$SHAPE_CAL:
3
2.301834E+000 9.195991E-004 -5.357180E-008
```

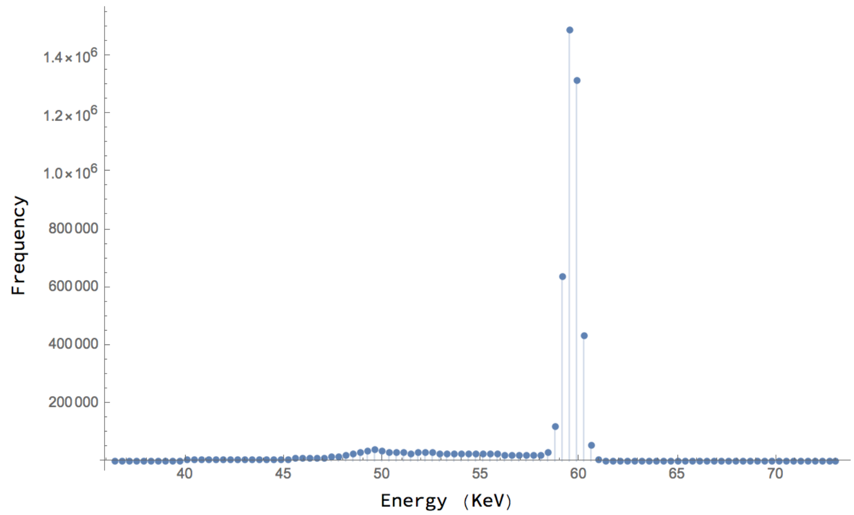


Figure 3.5

One Hour Run

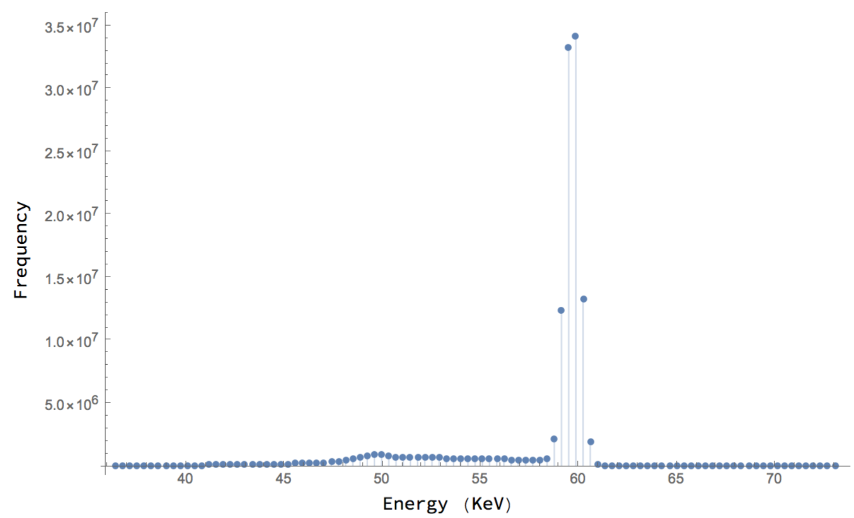


Figure 3.6

Twenty Four Hour Run

3.2.1 Physical measurement

The first step was to determine the mass attenuation coefficient of the alloy. This required X-ray measurement of the source only and with the thick Al foil, the thick Al foil and the actual density of the thickness of the Al foil. The mass of thick foil was measured using the high precision scale as shown in Figure 3.7. The mass was measured several times and the average determined as show in Table 3.2.

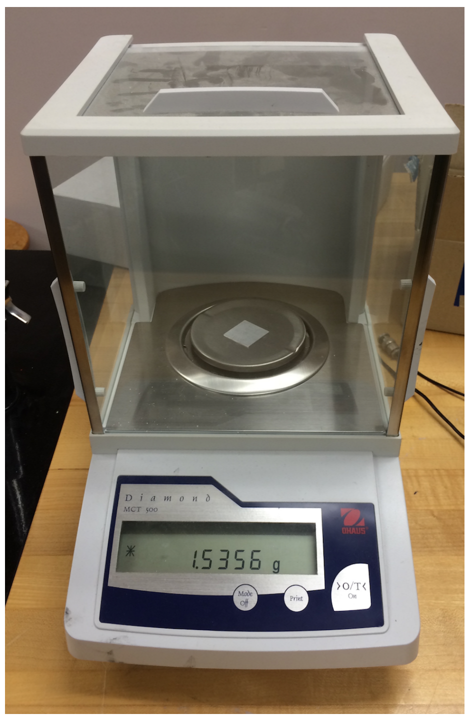


Figure 3.7

Scale

Table 3.2

Mass Measurements

Trial	Mass (g)
1	1.5354
2	1.5354
3	1.5358
4	1.5354
5	1.5356
6	1.5354
7	1.5354
8	1.5356
9	1.5354
10	1.5354
11	1.5353
12	1.5352
13	1.5354
14	1.5352
15	1.5356
Average	1.53543(14)

The thickness of thick foil was determined using a high precision micrometer. The thickness was measured several times as shown in Figure 3.8. Between each measurement, the micrometer needed to be recalibratd so, a second measurement was used to calibrate the micrometer. Summary of thickness data is shown in Table 3.3.

Figure 3.8

High Precision Micrometer

To get the areal density, the mass and the area of the foil were required. The area of thick foil was measured using a high accuracy toolmaker's microscope. The area was measured by tracing a polygon between the top and the bottom using the crosshair. Figure 3.9 shows the toolmaker's microscope. The area was calculated using :

Table 3.3

Thick foil measurements

Trial	Data 1 Thickness (mm)	Data 2 Thickness (mm)
1	0.8856	0.8838
2	0.8712	0.8722
3	0.8936	0.8909
4	08712	0.8722
5	0.8849	0.8868
6	0.8831	0.8891
7	0.8946	0.8923
8	0.8916	0.8919
9	0.8880	0.8884
10	0.8909	0.8885
11	0.8936	0.8909
12	0.8884	0.8851
13	0.8884	0.8851
14	0.8831	0.8891
15	0.8927	0.8844
Average	0.887(6)	0.885(6)

$$A = \frac{1}{2} \sum_{k=1}^{n-1} \left(x_{k+1} + x_k\right) \left(y_{k+1} - y_k\right) \tag{3.1}$$

The front area is $6.17067(4)$ cm^2, and the back area is $6.17060(4)$ cm^2. The weighted average of the two sides yielded a measurement of $6.17063(28)$ cm^2.

Figure 3.9

Toolmaker's Microscope

CHAPTER 4

DATA ANALYSIS

The exported data file from the germanium detector needed to be analyzed. The data file consists of 8192 values. Each value represents channel number, frequency rate of the photons and energy. There are numerous peaks in the data, the most prominent is the peak at 60 keV.

4.1 Data Modeling

The data needed to be corrected for the detector deadtime. The deadtime "Dt" can explained as the lost time as the detector was recording a pervious event. Events that occur during this "lost time" are not recorded, hence the data need to be corrected as fellowing:

$$\mathrm{Dt} = \frac{\mathrm{Lt} - \mathrm{Rt}}{\mathrm{Rt}} \tag{4.1}$$

where Dt is the dead time, Lt is the live time, and Rt is the real time. The data were modeled using Mathematica's by using the `NonlinearModelFit` command.

4.1.1 Gaussian function

Gaussian distribution is a continuous symmetric distribution. After the measurement, the data needed to be analyzed to determine the number of events or counts due to 60 keV x-rays from the source. The data were binned in energy bins of width 0.3666 keV. There were two primary components to the spectrum: the background and the peak. Since the data shows a bell shape curve a Gaussian model was used to fit the peak. According to the formula below :

$$f(x; p, \mu, \sigma) = p\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{4.2}$$

where p is the height of the peak, μ is the center, and σ is the characteristic width.

The background was fitted to several overlapping Gaussian functions. Mathematically:

$$f(x) = \sum_{i=1}^{n} f_i(x) \tag{4.3}$$

$$\text{where } f_i(x; p_i, \mu_i, \sigma_i) = p_i\, e^{-\frac{1}{2}\left(\frac{x-\mu_i}{\sigma_i}\right)^2} \tag{4.4}$$

where n is the number of peaks . The parameters are μ_i, σ_i, and p_i for $i = 1, \ldots, n$. $f_n(x)$ represent the peak while $f_i(x)$ for $i = 1, \ldots, n-1$ represent the background. A graph of the Gaussian fits along with the fit residuals are shown in.

4.2 Result

The Integrated counts "I_p" can be determined by subtracting the background from the peak and integrating according to : keV

$$I_p = \int_{-\infty}^{\infty} f_p(x)dx = p_p \sigma_p \sqrt{2\pi} \tag{4.5}$$

$$\approx \int_{\mu-2FWHM}^{\mu+2FWHM} f_p(x)dx \tag{4.6}$$

where $FWHM$ is the full width at half maximum which given by :

$$FWHM = 2\sigma\sqrt{2\pi}. \tag{4.7}$$

Note: the statistical error on the integrated counts in the peak is :

$$\sigma_p^2 \quad = \quad \sigma_t^2 + \sigma_{bg}^2 \tag{4.8}$$

where, p corresponds to the peak, bg corresponds to the background, and t corresponds to the total (peak + background). Since this is counting experiment, Poisson statistics apply. Hence $\sigma_t^2 \approx N_t$, $\sigma_p^2 \approx N_p$, $\sigma_{bg}^2 \approx N_{bg}$, where N_t, N_p, N_{bg} are correspond to the number of counts in the 60 keV peak, the background and the total respectively. Figure 4.1 and Figure 4.2 show Gaussian fits for 1 hr and 24 hr runs respectively. In addition,Table 4.1 show intensity calculation for integration range within 2σ for 1hr and 24hr. The Raw counts analysis was preformed for 1hr and 24hr data as show inTable 4.2.

Table 4.1

Intensity measurements

Trial	Gaussian fit for 1hr	Gaussian fit for 24hr
Source	$3.98(2) \times 10^6$	$9.60(7) \times 10^7$
Thin	$3.94(3) \times 10^6$	$9.50(8) \times 10^7$
Thick	$3.61(4) \times 10^6$	$8.73(9) \times 10^7$

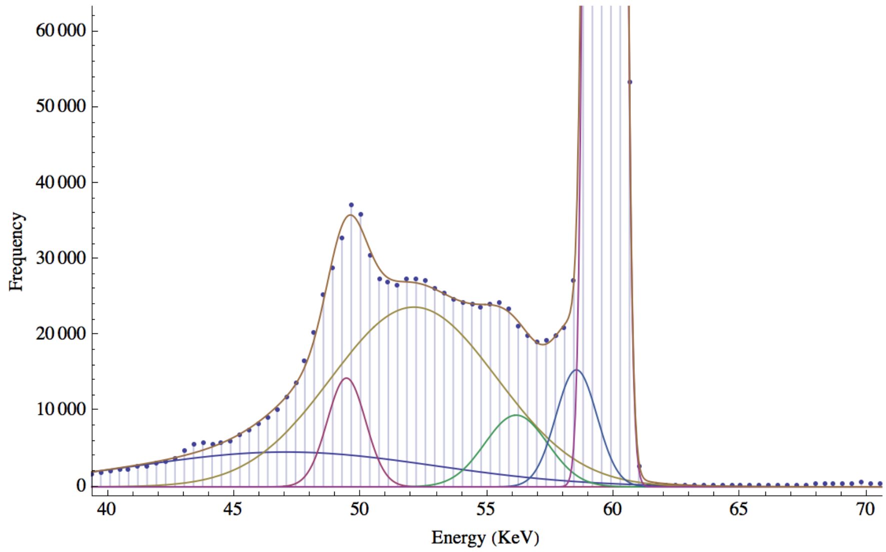


Figure 4.1

Gaussian fit for one hour run

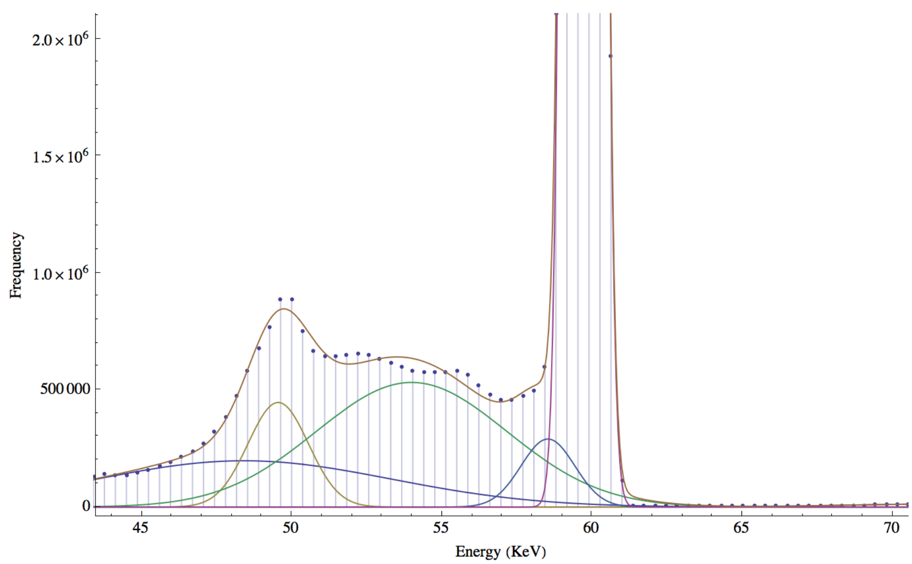


Figure 4.2

Gaussian fit for 24 hour run

Table 4.2

Raw Counts

Trial	1hr data	24hr data
Source	4.06×10^6	9.78×10^7
Thin	4.02×10^6	9.68×10^7
Thick	3.70×10^6	8.89×10^7

4.2.1 Attenuation Coefficients

The thickness of the thin foil was determined using the result of the attenuation coefficient μ from the thick foil measuerment. The mass attenuation coefficient at 60 keV was determined using equation 2.5, In other word:

$$\frac{\mu}{\rho} = \frac{\ln \frac{(I_0)}{(I_{tk})}}{M/A} \tag{4.9}$$

where I_{tk} is the intensity of the thick foil, I_0 is the intensity of the source, M is the mass of the thick foil, and A is the area of the thick foil. Similarly, the uncertainty is determined using

$$\sigma_\mu^2 = \sigma_x^2 \left(\frac{\partial \mu}{\partial x}\right)^2 + \sigma_{I_k}^2 \left(\frac{\partial \mu}{\partial I_k}\right)^2 + \sigma_{I_0}^2 \left(\frac{\partial \mu}{\partial I_0}\right)^2 \tag{4.10}$$

where σ_μ is the uncertainty associated with the attenuation coefficients, σ_x is the uncertainty associated with the Intensity of thin foil, σ_{I_k} is the uncertainty associated with the Intensity of thick foil, and σ_{I_0} is the uncertainty associated with the Intensity of the source only.

4.2.2 Monte Carlo Calculation

In addition, the theoretical value for the mass attenuation coefficient was calculated using a Monte Carlo method. In fact, the aluminum alloy AL 7075-T6 is composed of deferent elements as shown in Table 4.3. The NIST value for the attenuation coefficient for different elements at 60 keV is shown inTable 4.4

Table 4.3

Percent weight for each element in AL7075-T6[3]

	Al	Si	Fe	Cu	Mn	Mg	Cr	Zn	Ti	Ni
min	87.1	0.0	0.0	1.2	0.0	2.1	0.18	5.1	0.0	0.00
max	91.4	0.4	0.5	2.0	0.3	2.9	0.28	6.1	0.2	0.05

Table 4.4

Attenuation coefficients for elements in AL7075-T6[5]

	Al	Si	Fe	Cu	Mn	Mg	Cr	Zn	Ti	Ni
μ	0.2778	0.3207	1.205	1.593	1.060	0.2570	0.9639	1.760	0.7661	1.512

A total of 5,000,000 trials were generated, each trail, a mass attenuation coefficient was calculated using a weighted average of the mass attenuation coefficients of each component of alloys as following:

$$\mu = \frac{\sum_i m_i \mu_i}{\sum_i m_i} \tag{4.11}$$

where m_i is the mass of each component and μ_i is the mass attenuation coefficient of the component.. Next, A Gaussian distribution was fitted into the histogram data as shown in Figure 4.3. The mean value was determined to be 0.3874(52) $\frac{cm^2}{g}$.

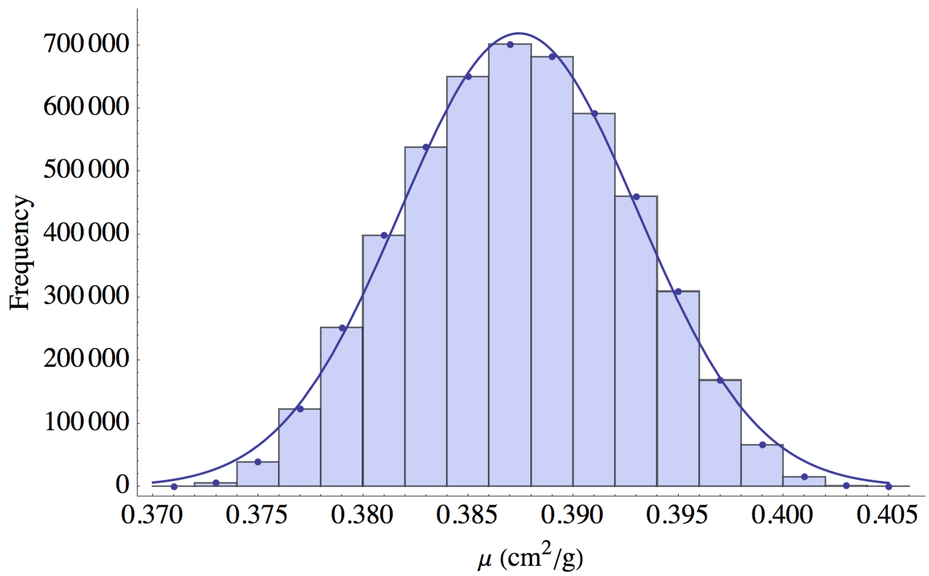


Figure 4.3

Monte Carlo calculation for AL7075-T6

4.2.3 Thickness of Thin Foil

The thickness was calculated using:

$$t_{th} = \frac{-\ln\frac{(I_{th})}{(I_0)}}{\mu} = x_k \frac{\ln\frac{(I_{th})}{(I_0)}}{\ln\frac{(I_k)}{(I_0)}} \tag{4.12}$$

where I_0 is the intensity of the source, I_{th} is the intensity with the thin foil. I_k is the intensity with the thick foil Similarly, the error in thickness was calculated using :

$$\sigma_{x_k}^2 = \sigma_{I_0}^2 \left(\frac{\partial_{x_k}}{\partial_{I_0}}\right)^2 + \sigma_{I_k}^2 \left(\frac{\partial_{x_k}}{\partial I_k}\right)^2 + \sigma_{I_{th}} \left(\frac{\partial_{x_k}}{\partial I_{th}}\right)^2 + \sigma_x^2 \left(\frac{\partial_{x_k}}{\partial x}\right)^2 \quad (4.13)$$

where, $\sigma_{x_k}^2$ is the error in the thickness, $\sigma_{I_{th}}$ is the error in the intensity of thin foil.

The fitted data from table 4.1 suggest that the experimental value of mass attenuation coefficient is $0.3853 \pm 0.0003\frac{cm^2}{g}$ for one hour run and $0.3815 \pm 0.0008\frac{cm^2}{g}$ for the twenty four hour run. In addition, the thickness is $3.78 \pm 0.30(mil)$ for one hour run and $3.73 \pm 0.1(mil)$ for the twenty four hour run.

In comparison with the Raw count analysis, it shows that the experimental value of mass attenuation coefficient is $0.3755 \pm 0.0029\frac{cm^2}{g}$ for one hour run and $0.3818 \pm 0.0008\frac{cm^2}{g}$ for the twenty four hour run. In addition, the thickness is $3.87 \pm 0.25(mil)$ for one hour run and $3.83 \pm 0.05(mil)$ for the twenty four hour run.

CHAPTER 5

CONCLUSIONS

5.1 Conclusion

In this experiment, we measure the thickness of thin foil using x-ray attenuation. The experimental value of mass attenuation coefficient is $0.03818 \pm 0.0008 \frac{cm^2}{g}$ when compare to Monto Carlo method yield very accurate measurements.

The measurements of the thickness using x-ray attenuation yield very precise measurement with high accuracy. Experimentally, the thickness determined to be $3.83 \pm 0.05(mil)$.

A potential source of uncertianity that can affect the result such as removal and placement of the radioactive material between each run. Also, systemic error due to the way that detector designed. In addition, the source hold was not secure enough which may cause the vibration of the source holder to produce inconsistencies.

5.2 Further Research

Measure the material thickness using x-ray attenuation has proven to be accurate. Furthermore, the design of the apparatus should be improved by adding another axes which will allow two two dimensional scan. In addition, one should consider using x-rays attenuation to measure the thickness for different materials such as gold and lead. Most important, one should consider creating GEANT simulation of the experiment which will help to explain the background of the spectrum and possibly help optimize the setup.

REFERENCES

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APPENDIX A

MATHEMATICA ANALYSIS

A.1 Mathematica Analysis

The following pages include the Mathematica code that used to for data analysis

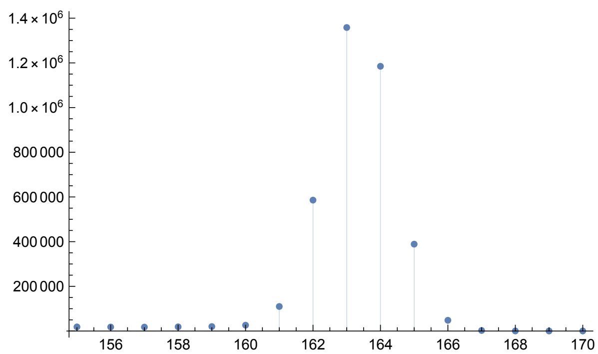
```
ClearAll["Global`*"]
(*~ MacOS ~*)
    main="/Users/abdullahaltayyer/Documents/Research/newdata";

  SetDirectory[main];
  $Line=0;
```

ThickFilm

```
imported = Import["Am-241-ThickFilm-1hour.spe", "Data"];
times = ToExpression[StringSplit[imported[[10]]]];
deadtime = N[Abs[times[[1]] - times[[2]]] / times[[2]]];
imported[[ ;; 15]] // TableForm
imported[[-16 ;;]] // TableForm
freq = ToExpression[imported[[13 ;; -15]]] / (1 - deadtime);
thickdata[1] = Table[{i, freq[[i]]}, {i, Length[freq]}];
ListPlot[thickdata[1][[155 ;; 170]], PlotRange → All, Filling → Bottom]
```

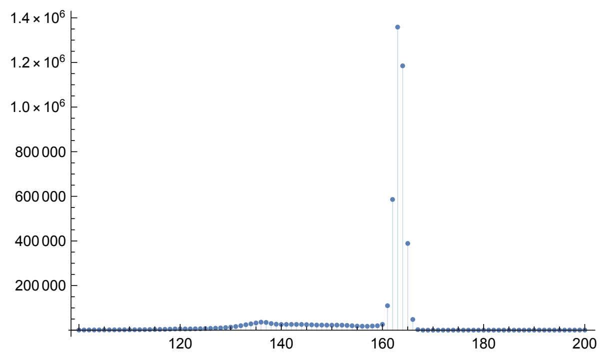
```
$SPEC_ID:
No sample description was entered.
$SPEC_REM:
DET# 1
DETDESC# HPGe Detector 1
AP# GammaVision Version 6.07
$DATE_MEA:
11/27/2007 21:52:37
$MEAS_TIM:
3600 3786
$DATA:
0 8191
       0
       0
       0
```

```
       0
       0
$ROI:
0
$PRESETS:
Live Time
3600
0
$ENER_FIT:
0.137869 0.366631
$MCA_CAL:
3
1.378689E-001 3.666307E-001 -4.415950E-008 keV
$SHAPE_CAL:
3
2.301834E+000 9.195991E-004 -5.357180E-008
```

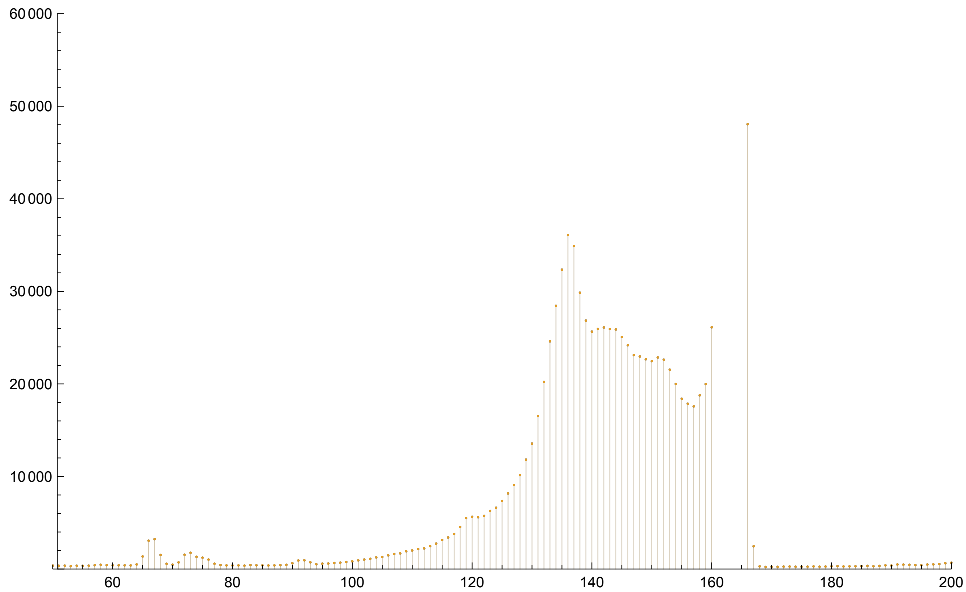


EQ

```
F[x_, mu_, sigma_, a_] := a * Exp[- 1/2 * ((x - mu)/sigma)^2]

F2[x_, mu_, sigma_, a_] := a * Exp[- 1/2 * ((x - mu)/sigma)^2]

G[x_, mu_, sigma_, a_] := Sum[F[x, mu[[i]], sigma[[i]], a[[i]]], {i, 1, Length[mu]}]

chisq2[data_, mu_, sigma_, a_] :=
 Sum[(data[[i, 2]] - G[data[[i, 1]], mu, sigma, a])^2, {i, Length[data]}]
backgroundfunction[x_, mu1_, mu2_, mu3_, mu4_, sigma1_, sigma2_,
   sigma3_, sigma4_, a1_, a2_, a3_, a4_] := F2[x, mu1, sigma1, a1] +
   F2[x, mu2, sigma2, a2] + F2[x, mu3, sigma3, a3] + F2[x, mu4, sigma4, a4];
fwhm[sigma_] := 2 * sigma * Sqrt[2 * Log[2]];

err[F_, w__] := {F[Map[First, List[w]] /. List → Sequence],
  Block[{parms = Table[Unique[], {x, 1, Length[List[w]]}],
    values = Table[List[w][[i, 1]], {i, 1, Length[List[w]]}],
    errors = Table[List[w][[i, 2]], {i, 1, Length[List[w]]}]},
   Sqrt[Total[Table[(D[F[parms /. List → Sequence], parms[[i]]] * errors[[i]]) ^2,
       {i, 1, Length[values]}] /.
      Table[parms[[i]] → values[[i]], {i, 1, Length[values]}]]]]}

ListPlot[thickdata[1][[100 ;; 200]], Filling → Bottom, PlotRange → All]
```

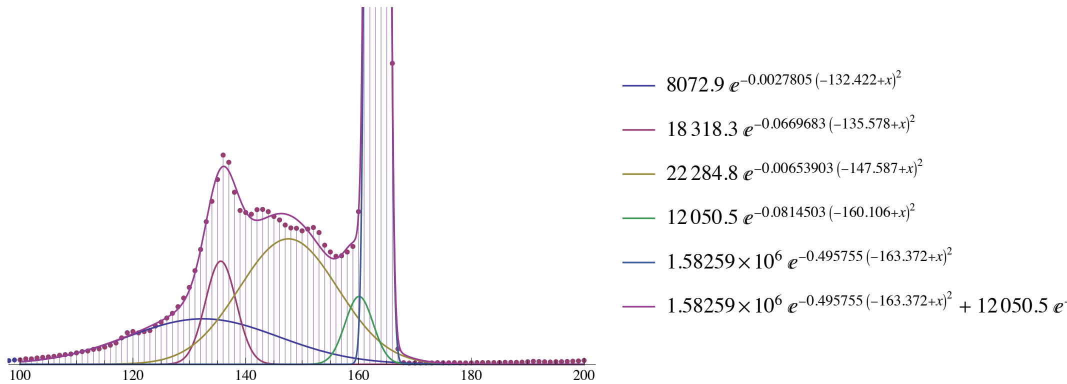


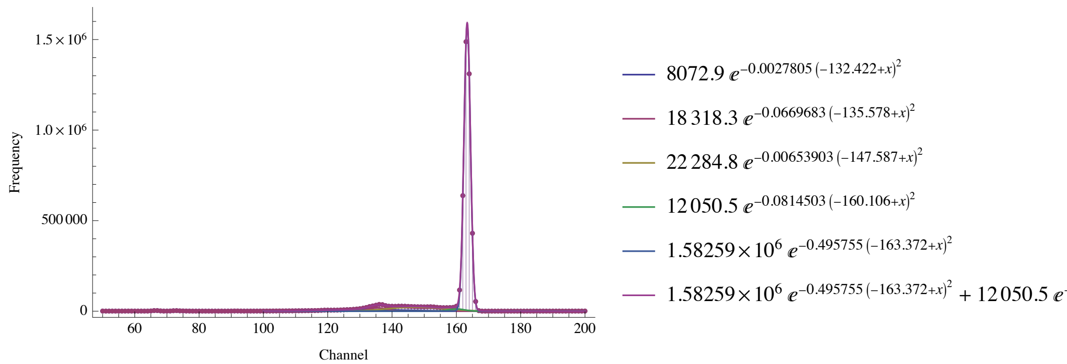
```
bkgdata = thickdata[1];
viewingdata = thickdata[1][[50 ;; 200]];
ListPlot[{viewingdata, bkgdata},
 Filling → Bottom, PlotRange → {{50, 200}, {0, 60 000}}]
```



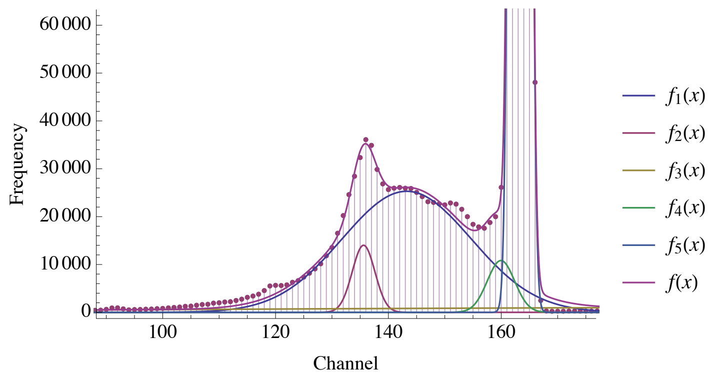
```
mu = {141.3, 135.9, 151.3, 160.3, 163.3};
sigma = {14.1, 1.8, 3.8, 2.1, 1.0};
a = {21 500, 20 000, 16 000, 20 000, 1 400 000}; n = Length[mu];

model = NonlinearModelFit[bkgdata, G[x, Table[Symbol["mu" <> ToString[i]], {i, 1, n}],
     Table[Symbol["sigma" <> ToString[i]], {i, 1, n}],
     Table[Symbol["a" <> ToString[i]], {i, 1, n}]],
    Join[Table[{Symbol["mu" <> ToString[i]], mu[[i]]}, {i, 1, n}],
     Table[{Symbol["sigma" <> ToString[i]], sigma[[i]]}, {i, 1, n}],
     Table[{Symbol["a" <> ToString[i]], a[[i]]}, {i, 1, n}]], x];
model["ParameterTable"]
params = model["BestFitParameters"];
paramerrors = model["ParameterErrors"];

theme = "Classic";
Show[ListPlot[{viewingdata, bkgdata[[100 ;; 200]]},
   Filling → Bottom, PlotTheme → theme, PlotRange → All], Plot[Evaluate@
    Join[Table[F[x, params[[i, 2]], params[[i + n, 2]], params[[i + 2 n, 2]]], {i, 1, n}],
     {G[x, params[[ ;; n, 2]], params[[n + 1 ;; 2 n, 2]], params[[2 n + 1 ;; 3 n, 2]]]}],
   {x, 100, 200}, PlotRange → All, PlotTheme → theme, PlotLegends → "Expressions"],
  PlotRange → {{100, 200}, {0, 60 000}}]


Show[ListPlot[{viewingdata, bkgdata[[50 ;; 200]]},
   Filling → Bottom, PlotTheme → theme, PlotRange → All], Plot[Evaluate@
    Join[Table[F[x, params[[i, 2]], params[[i + n, 2]], params[[i + 2 n, 2]]], {i, 1, n}],
     {G[x, params[[ ;; n, 2]], params[[n + 1 ;; 2 n, 2]], params[[2 n + 1 ;; 3 n, 2]]]}],
   {x, 100, 200}, PlotRange → All, PlotTheme → theme, PlotLegends → "Expressions"],
  Frame → {{True, False}, {True, False}}, FrameTicks → {{True, False}, {True, False}},
  FrameLabel → {"Channel", "Frequency"}, PlotRange → {{50, 200}, All}]
```

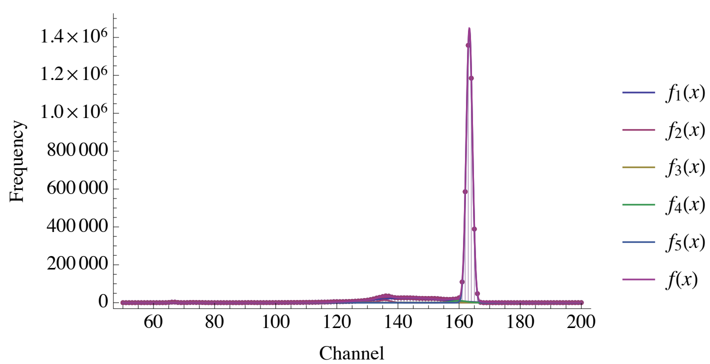
	Estimate	Standard Error	t-Statistic	P-Value
mu1	132.422	3.82905	34.5834	1.01101×10^{-244}
mu2	135.578	0.0365532	3709.06	$4.5533288187 \times 10^{-13193}$
mu3	147.587	0.271831	542.936	$9.3079411364 \times 10^{-6417}$
mu4	160.106	0.155122	1032.13	$5.0677196522 \times 10^{-8663}$
mu5	163.372	0.000240765	678555.	$5.2074038168 \times 10^{-31691}$
sigma1	13.4098	1.42445	9.41402	6.09329×10^{-21}
sigma2	2.73244	0.046059	59.3247	$4.0901835474 \times 10^{-638}$
sigma3	8.74436	0.440779	19.8384	1.39768×10^{-85}
sigma4	2.47764	0.154786	16.0069	8.29275×10^{-57}
sigma5	1.00427	0.000420221	2389.87	$1.20167006071 \times 10^{-11633}$
a1	8072.9	1524.75	5.29458	1.2239×10^{-7}
a2	18318.3	314.479	58.2495	$8.1800055399 \times 10^{-619}$
a3	22284.8	2702.14	8.24709	1.87582×10^{-16}
a4	12050.5	450.839	26.7291	5.75738×10^{-151}
a5	1.58259×10^{6}	840.023	1883.98	$1.56049491087 \times 10^{-10790}$


8072.9 e^−0.0027805 (−132.422+x)^2
18 318.3 e^−0.0669683 (−135.578+x)^2
22 284.8 e^−0.00653903 (−147.587+x)^2
12 050.5 e^−0.0814503 (−160.106+x)^2
1.58259×10^6 e^−0.495755 (−163.372+x)^2
1.58259×10^6 e^−0.495755 (−163.372+x)^2 + 12 050.5 e
100
120
140
160
180
200


1.5×10^6
1.0×10^6
500 000
0
Frequency
60
80
100
120
140
160
180
200
Channel
8072.9 e^−0.0027805 (−132.422+x)^2
18 318.3 e^−0.0669683 (−135.578+x)^2
22 284.8 e^−0.00653903 (−147.587+x)^2
12 050.5 e^−0.0814503 (−160.106+x)^2
1.58259×10^6 e^−0.495755 (−163.372+x)^2
1.58259×10^6 e^−0.495755 (−163.372+x)^2 + 12 050.5 e

```
Show[ListPlot[{viewingdata, bkgdata[[50 ;; 200]]},
   Filling → Bottom, PlotTheme → theme, PlotRange → All], Plot[Evaluate@
    Join[Table[F[x, params[[i, 2]], params[[i + n, 2]], params[[i + 2 n, 2]]], {i, 1, n}],
     {G[x, params[[ ;; n, 2]], params[[n + 1 ;; 2 n, 2]], params[[2 n + 1 ;; 3 n, 2]]]}],
   {x, 50, 200}, PlotRange → All, PlotTheme → theme,
   PlotLegends → {"f1(x)", "f2(x)", "f3(x)", "f4(x)", "f5(x)", "f(x)"}],
  PlotRange → {{90, 175}, {0, 60 000}}, Frame → {{True, False}, {True, False}},
  FrameTicks → {{True, False}, {True, False}},
  FrameLabel → {"Channel", "Frequency"}, BaseStyle → {FontSize → 12}]
Show[ListPlot[{viewingdata, bkgdata[[50 ;; 200]]}, Filling → Bottom,
   PlotTheme → theme, PlotRange → All], Plot[Evaluate@
    Join[Table[F[x, params[[i, 2]], params[[i + n, 2]], params[[i + 2 n, 2]]], {i, 1, n}],
     {G[x, params[[ ;; n, 2]], params[[n + 1 ;; 2 n, 2]], params[[2 n + 1 ;; 3 n, 2]]]}],
   {x, 50, 200}, PlotRange → All, PlotTheme → theme,
   PlotLegends → {"f1(x)", "f2(x)", "f3(x)", "f4(x)", "f5(x)", "f(x)"}],
  PlotRange → {{50, 200}, All}, Frame → {{True, False}, {True, False}},
  FrameTicks → {{True, False}, {True, False}},
  FrameLabel → {"Channel", "Frequency"}, BaseStyle → {FontSize → 12}]
```





```
Table[{params[[i, 2]], paramerrors[[i]], 100 * paramerrors[[i]] / params[[i, 2]]},
  {i, {5, 10, 15}}] // TableForm
background = Table[err[F2, {i[[1]], 0}, {params[[5, 2]], paramerrors[[5]]},
    {params[[10, 2]], paramerrors[[10]]},
    {params[[15, 2]], paramerrors[[15]]}], {i, bkgdata}];
counts[3] = Total[background[[All, 1]]];
countserror[3] = Sqrt[Total[(background[[All, 2]])^2]];
{counts[3], countserror[3], 100 countserror[3] / counts[3]}
```

163.364	0.000232785	0.000142494
1.00416	0.000360888	0.0359392
1.43801×10^6	614.841	0.0427564

$\{3.61956 \times 10^6, 1061.25, 0.0293198\}$

```
peakbackground =
 Apply[err, Join[{backgroundfunction, {params[[5, 2]], paramerrors[[5]]}},
   Table[{params[[i, 2]], paramerrors[[i]]}, {i, 1, 4}],
   Table[{params[[i + 5, 2]], paramerrors[[i]]}, {i, 1, 4}],
   Table[{params[[i + 10, 2]], paramerrors[[i]]}, {i, 1, 4}]]]
countsalt[3] = NIntegrate[F2[x, params[[5, 2]], params[[5 + 5, 2]],
    params[[5 + 10, 2]]], {x, params[[5, 2]] - 2 * fwhm[params[[5 + 5, 2]]],
    params[[5, 2]] + 2 * fwhm[params[[5 + 5, 2]]]}];
countserroralt[3] = Sqrt[paramerrors[[-1]]^2 + peakbackground[[2]]^2];
{countsalt[3], countserroralt[3], 100 * countserroralt[3] / countsalt[3]}
```

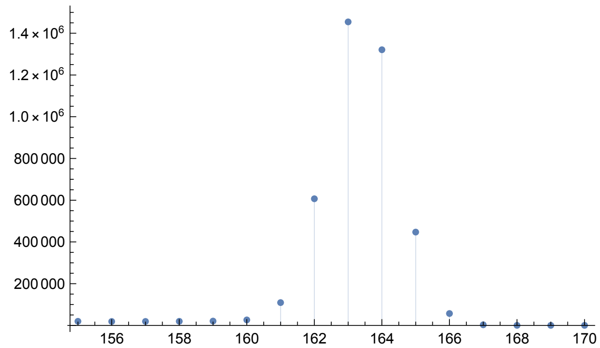
{10 294.7, 524.066}

$\{3.61956 \times 10^6, 807.883, 0.02232\}$

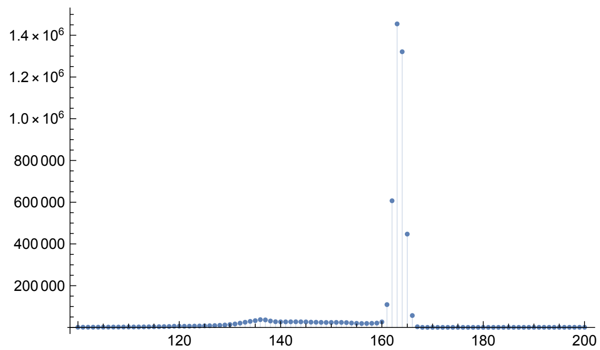
Thin Film

```
imported = Import["Am-241-ThinFilm-1hour.spe", "Data"];
times = ToExpression[StringSplit[imported[[10]]]];
deadtime = N[Abs[times[[1]] - times[[2]]] / times[[2]]];
imported[[ ;; 15]] // TableForm
imported[[-16 ;;]] // TableForm
freq = ToExpression[imported[[13 ;; -15]]] / (1 - deadtime);
thindata[1] = Table[{i, freq[[i]]}, {i, Length[freq]}];
ListPlot[thindata[1][[155 ;; 170]], PlotRange → All, Filling → Bottom]
```

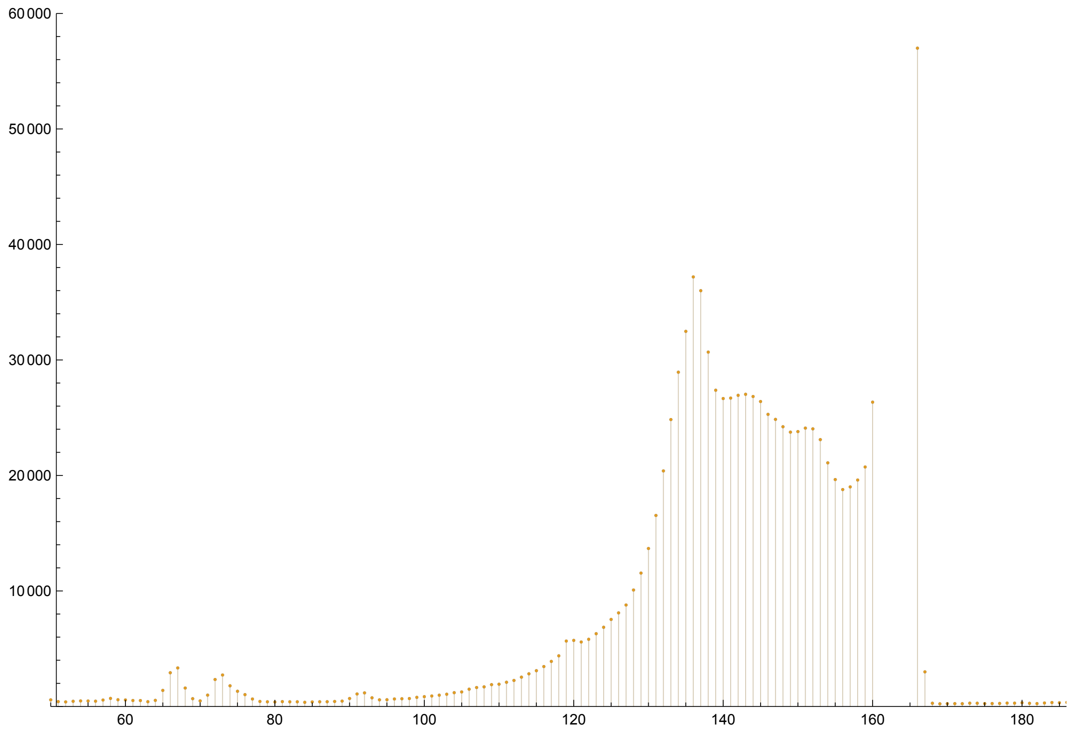
```
$SPEC_ID:
No sample description was entered.
$SPEC_REM:
DET# 1
DETDESC# HPGe Detector 1
AP# GammaVision Version 6.07
$DATE_MEA:
11/27/2007 19:16:54
$MEAS_TIM:
3600 3799
$DATA:
0 8191
       0
       0
       0

       0
       1
$ROI:
0
$PRESETS:
Live Time
3600
0
$ENER_FIT:
0.137869 0.366631
$MCA_CAL:
3
1.378689E-001 3.666307E-001 -4.415950E-008 keV
$SHAPE_CAL:
3
2.301834E+000 9.195991E-004 -5.357180E-008
```




```
ListPlot[thindata[1][[100 ;; 200]], Filling → Bottom, PlotRange → All]
```



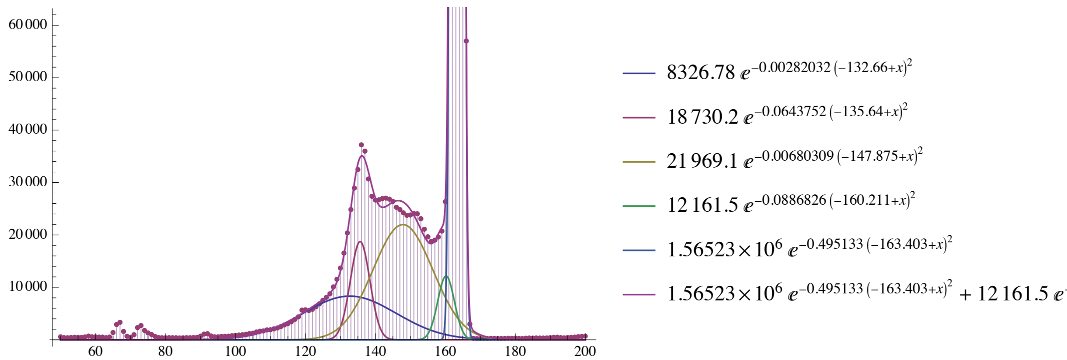
```
bkgdata = thindata[1];
viewingdata = thindata[1][[50 ;; 200]];
ListPlot[{viewingdata, bkgdata},
 Filling → Bottom, PlotRange → {{50, 200}, {0, 60 000}}]
```

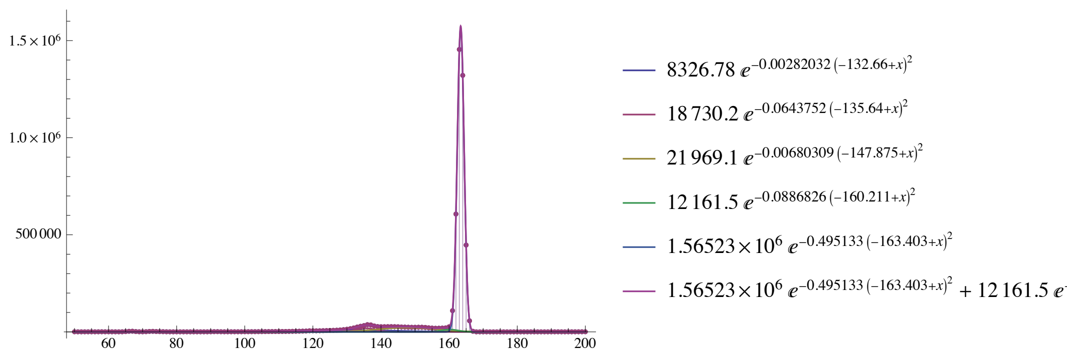


```
mu = {119.6, 136, 142, 151, 162.2}; sigma = {14.6, 3.8, 3.4, 4.0, 1.0};
a = {5800, 36 000, 27 400, 27 400, 730 000}; n = Length[mu];
model =
  NonlinearModelFit[bkgdata, G[x, Table[Symbol["mu" <> ToString[i]], {i, 1, n}],
    Table[Symbol["sigma" <> ToString[i]], {i, 1, n}],
    Table[Symbol["a" <> ToString[i]], {i, 1, n}]],
   Join[Table[{Symbol["mu" <> ToString[i]], mu[[i]]}, {i, 1, n}],
    Table[{Symbol["sigma" <> ToString[i]], sigma[[i]]}, {i, 1, n}],
    Table[{Symbol["a" <> ToString[i]], a[[i]]}, {i, 1, n}]], x];
model["ParameterTable"]
params = model["BestFitParameters"];
paramerrors = model["ParameterErrors"];

theme = "Classic";
Show[ListPlot[{viewingdata, bkgdata[[50 ;; 200]]},
  Filling → Bottom, PlotTheme → theme, PlotRange → All], Plot[Evaluate@
   Join[Table[F[x, params[[i, 2]], params[[i + n, 2]], params[[i + 2 n, 2]]], {i, 1, n}],
    {G[x, params[[ ;; n, 2]], params[[n + 1 ;; 2 n, 2]], params[[2 n + 1 ;; 3 n, 2]]]}],
  {x, 50, 200}, PlotRange → All, PlotTheme → theme, PlotLegends → "Expressions"],
 PlotRange → {{50, 200}, {0, 60 000}}]
Show[ListPlot[{viewingdata, bkgdata[[50 ;; 200]]}, Filling → Bottom,
  PlotTheme → theme, PlotRange → All], Plot[Evaluate@
   Join[Table[F[x, params[[i, 2]], params[[i + n, 2]], params[[i + 2 n, 2]]], {i, 1, n}],
    {G[x, params[[ ;; n, 2]], params[[n + 1 ;; 2 n, 2]], params[[2 n + 1 ;; 3 n, 2]]]}],
  {x, 50, 200}, PlotRange → All, PlotTheme → theme, PlotLegends → "Expressions"],
 PlotRange → {{50, 200}, All}]
```

... NonlinearModelFit: Failed to converge to the requested accuracy or precision within 100 iterations.

	Estimate	Standard Error	t-Statistic	P-Value
mu1	132.66	3.32601	39.8857	$4.8192814368\times10^{-318}$
mu2	135.64	0.0368144	3684.43	$2.03172845790\times10^{-13169}$
mu3	147.875	0.242294	610.311	$3.8068412771\times10^{-6822}$
mu4	160.211	0.158132	1013.15	$1.25293189680\times10^{-8597}$
mu5	163.403	0.000256023	638 237.	$1.77496626073\times10^{-31473}$
sigma1	13.3148	1.26682	10.5104	1.12696×10^{-25}
sigma2	2.78693	0.0449575	61.9902	$1.49935701747\times10^{-686}$
sigma3	−8.57298	0.401987	−21.3265	2.94996×10^{-98}
sigma4	2.37446	0.151533	15.6697	1.50781×10^{-54}
sigma5	1.0049	0.000431907	2326.67	$1.32796233305\times10^{-11538}$
a1	8326.78	1332.63	6.24839	4.35459×10^{-10}
a2	18 730.2	305.478	61.3146	$3.3808317561\times10^{-674}$
a3	21 969.1	2414.03	9.10061	1.11186×10^{-19}
a4	12 161.5	438.459	27.7369	6.5615×10^{-162}
a5	1.56523×10^{6}	895.176	1748.51	$3.4794926160\times10^{-10526}$






```
Table[{params[[i, 2]], paramerrors[[i]], 100 * paramerrors[[i]] / params[[i, 2]]},
  {i, {5, 10, 15}}] // TableForm
background = Table[err[F2, {i[[1]], 0}, {params[[5, 2]], paramerrors[[5]]},
    {params[[10, 2]], paramerrors[[10]]},
    {params[[15, 2]], paramerrors[[15]]}], {i, bkgdata}];
counts[4] = Total[background[[All, 1]]];
countserror[4] = Sqrt[Total[(background[[All, 2]])^2]];
{counts[4], countserror[4], 100 countserror[4] / counts[4]}
```

163.403	0.000256023	0.000156682
1.0049	0.000431907	0.04298
1.56523×10^6	895.176	0.0571915

$\{3.94268 \times 10^6, 1472.83, 0.0373561\}$

```
peakbackground =
 Apply[err, Join[{backgroundfunction, {params[[5, 2]], paramerrors[[5]]}},
   Table[{params[[i, 2]], paramerrors[[i]]}, {i, 1, 4}],
   Table[{params[[i + 5, 2]], paramerrors[[i]]}, {i, 1, 4}],
   Table[{params[[i + 10, 2]], paramerrors[[i]]}, {i, 1, 4}]]]
countsalt[4] = NIntegrate[F2[x, params[[5, 2]], params[[5 + 5, 2]],
    params[[5 + 10, 2]]], {x, params[[5, 2]] - 2 * fwhm[params[[5 + 5, 2]]],
    params[[5, 2]] + 2 * fwhm[params[[5 + 5, 2]]]}];
countserroralt[4] = Sqrt[paramerrors[[-1]] ^ 2 + peakbackground[[2]] ^ 2];
{countsalt[4], countserroralt[4],
   100 * countserroralt[4] / countsalt[4]} peakbackground =
 Apply[err, Join[{backgroundfunction, {params[[5, 2]], paramerrors[[5]]}},
   Table[{params[[i, 2]], paramerrors[[i]]}, {i, 1, 4}],
   Table[{params[[i + 5, 2]], paramerrors[[i]]}, {i, 1, 4}],
   Table[{params[[i + 10, 2]], paramerrors[[i]]}, {i, 1, 4}]]]
countsalt[4] = NIntegrate[F2[x, params[[5, 2]], params[[5 + 5, 2]],
    params[[5 + 10, 2]]], {x, params[[5, 2]] - 2 * fwhm[params[[5 + 5, 2]]],
    params[[5, 2]] + 2 * fwhm[params[[5 + 5, 2]]]}];
countserroralt[4] = Sqrt[paramerrors[[-1]] ^ 2 + peakbackground[[2]] ^ 2];
{countsalt[4], countserroralt[4], 100 * countserroralt[4] / countsalt[4]}
```

{9766.36, 1206.76}

... Set: Tag Times in {9766.36, 1206.76}{3.94267 × 10^6, 1502.54, 0.0381097} is Protected.

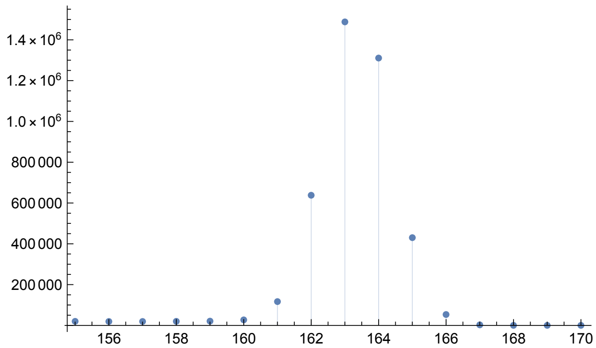
{9766.36, 1206.76}

{3.94267 × 10^6, 1502.54, 0.0381097}

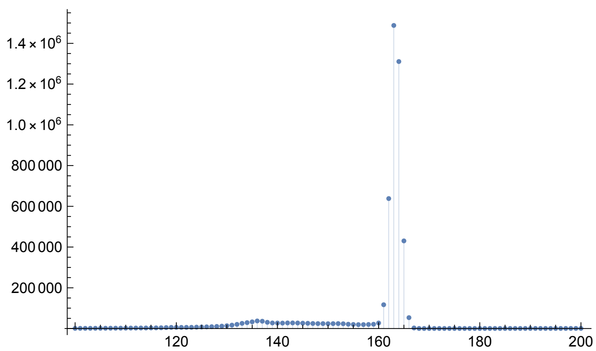
Source

```
imported = Import["Am-241-only-1hour.spe", "Data"];
times = ToExpression[StringSplit[imported[[10]]]];
deadtime = N[Abs[times[[1]] - times[[2]]] / times[[2]]];
imported[[ ;; 15]] // TableForm
imported[[-16 ;;]] // TableForm
freq = ToExpression[imported[[13 ;; -15]]] / (1 - deadtime);
sourcedata[1] = Table[{i, freq[[i]]}, {i, Length[freq]}];
ListPlot[sourcedata[1][[155 ;; 170]], PlotRange → All, Filling → Bottom]
```

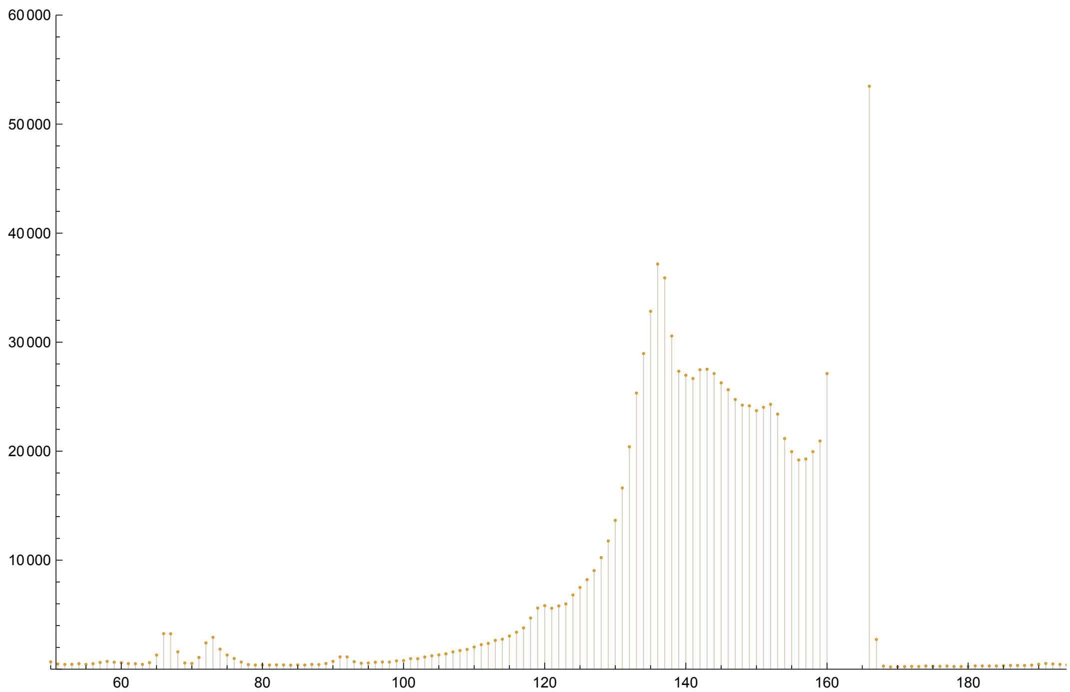
```
$SPEC_ID:
No sample description was entered.
$SPEC_REM:
DET# 1
DETDESC# HPGe Detector 1
AP# GammaVision Version 6.07
$DATE_MEA:
11/27/2007 18:05:09
$MEAS_TIM:
3600 3801
$DATA:
0 8191
       0
       0
       0

       0
       1
$ROI:
0
$PRESETS:
Live Time
3600
0
$ENER_FIT:
0.137869 0.366631
$MCA_CAL:
3
1.378689E-001 3.666307E-001 -4.415950E-008 keV
$SHAPE_CAL:
3
2.301834E+000 9.195991E-004 -5.357180E-008
```




```
ListPlot[sourcedata[1][[100 ;; 200]], Filling → Bottom, PlotRange → All]
```

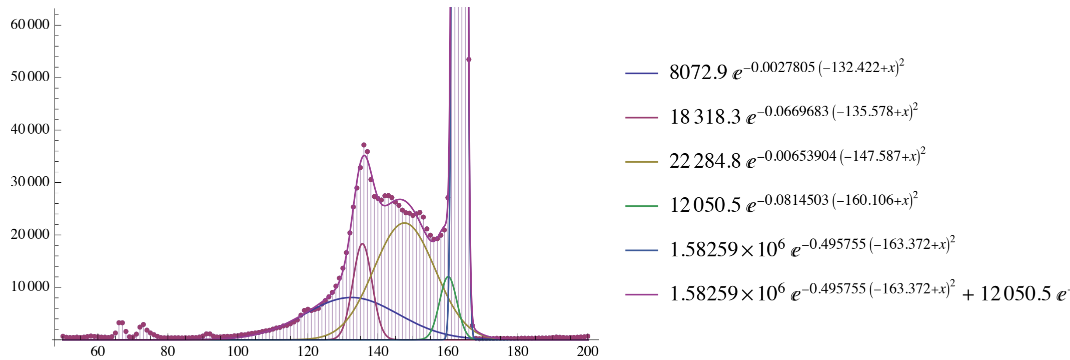


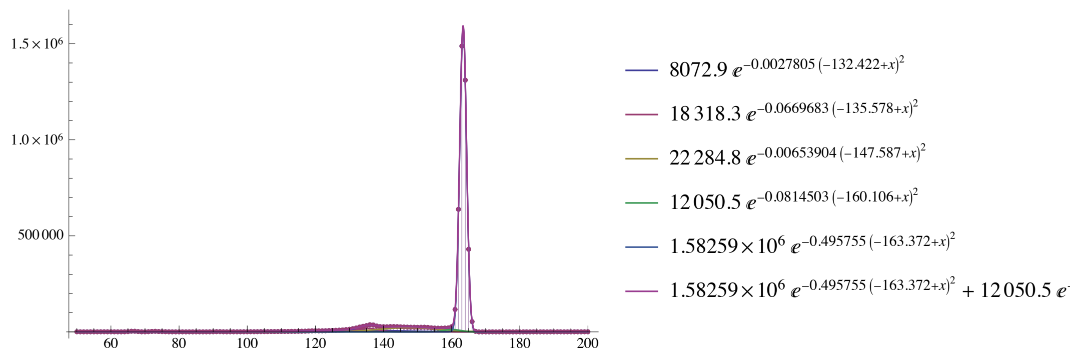
```
bkgdata = sourcedata[1];
viewingdata = sourcedata[1][[50 ;; 200]];
ListPlot[{viewingdata, bkgdata},
 Filling → Bottom, PlotRange → {{50, 200}, {0, 60000}}]
```



```
mu = {119.6, 136, 142, 151, 162.2}; sigma = {14.6, 3.8, 3.4, 4.0, 1.0};
a = {5800, 36000, 27400, 27400, 730000}; n = Length[mu];
model =
  NonlinearModelFit[bkgdata, G[x, Table[Symbol["mu" <> ToString[i]], {i, 1, n}],
    Table[Symbol["sigma" <> ToString[i]], {i, 1, n}],
    Table[Symbol["a" <> ToString[i]], {i, 1, n}]],
   Join[Table[{Symbol["mu" <> ToString[i]], mu[[i]]}, {i, 1, n}],
    Table[{Symbol["sigma" <> ToString[i]], sigma[[i]]}, {i, 1, n}],
    Table[{Symbol["a" <> ToString[i]], a[[i]]}, {i, 1, n}]], x];
model["ParameterTable"]
params = model["BestFitParameters"];
paramerrors = model["ParameterErrors"];

theme = "Classic";
Show[ListPlot[{viewingdata, bkgdata[[50 ;; 200]]},
  Filling → Bottom, PlotTheme → theme, PlotRange → All], Plot[Evaluate@
   Join[Table[F[x, params[[i, 2]], params[[i + n, 2]], params[[i + 2 n, 2]]], {i, 1, n}],
    {G[x, params[[ ;; n, 2]], params[[n + 1 ;; 2 n, 2]], params[[2 n + 1 ;; 3 n, 2]]]}],
  {x, 50, 200}, PlotRange → All, PlotTheme → theme, PlotLegends → "Expressions"],
 PlotRange → {{50, 200}, {0, 60000}}]
Show[ListPlot[{viewingdata, bkgdata[[50 ;; 200]]}, Filling → Bottom,
  PlotTheme → theme, PlotRange → All], Plot[Evaluate@
   Join[Table[F[x, params[[i, 2]], params[[i + n, 2]], params[[i + 2 n, 2]]], {i, 1, n}],
    {G[x, params[[ ;; n, 2]], params[[n + 1 ;; 2 n, 2]], params[[2 n + 1 ;; 3 n, 2]]]}],
  {x, 50, 200}, PlotRange → All, PlotTheme → theme, PlotLegends → "Expressions"],
 PlotRange → {{50, 200}, All}]
```

	Estimate	Standard Error	t-Statistic	P-Value
mu1	132.422	3.82905	34.5834	1.01116×10^{-244}
mu2	135.578	0.0365532	3709.06	$4.5540740319 \times 10^{-13193}$
mu3	147.587	0.271831	542.936	$9.2751514918 \times 10^{-6417}$
mu4	160.106	0.155122	1032.13	$5.0667118894 \times 10^{-8663}$
mu5	163.372	0.000240765	678555.	$5.2077694360 \times 10^{-31691}$
sigma1	13.4098	1.42445	9.41402	6.09319×10^{-21}
sigma2	2.73244	0.046059	59.3247	$4.0918857897 \times 10^{-638}$
sigma3	8.74436	0.44078	19.8384	1.39819×10^{-85}
sigma4	2.47764	0.154786	16.0069	8.29316×10^{-57}
sigma5	1.00427	0.000420221	2389.87	$1.20179537166 \times 10^{-11633}$
a1	8072.9	1524.75	5.29458	1.22391×10^{-7}
a2	18318.3	314.479	58.2495	$8.1884307790 \times 10^{-619}$
a3	22284.8	2702.14	8.24708	1.87588×10^{-16}
a4	12050.5	450.839	26.729	5.75889×10^{-151}
a5	1.58259×10^{6}	840.023	1883.98	$1.56091954800 \times 10^{-10790}$






```
Table[{params[[i, 2]], paramerrors[[i]], 100 * paramerrors[[i]] / params[[i, 2]]},
  {i, {5, 10, 15}}] // TableForm
background = Table[err[F2, {i[[1]], 0}, {params[[5, 2]], paramerrors[[5]]},
    {params[[10, 2]], paramerrors[[10]]},
    {params[[15, 2]], paramerrors[[15]]}], {i, bkgdata}];
counts[4] = Total[background[[All, 1]]];
countserror[4] = Sqrt[Total[(background[[All, 2]])^2]];
{counts[4], countserror[4], 100 countserror[4] / counts[4]}
```

163.372	0.000240765	0.000147372
1.00427	0.000420221	0.0418433
1.58259×10^6	840.023	0.053079

$\{3.98392\times10^6, 1402.17, 0.0351957\}$

```
peakbackground =
 Apply[err, Join[{backgroundfunction, {params[[5, 2]], paramerrors[[5]]}},
   Table[{params[[i, 2]], paramerrors[[i]]}, {i, 1, 4}],
   Table[{params[[i + 5, 2]], paramerrors[[i]]}, {i, 1, 4}],
   Table[{params[[i + 10, 2]], paramerrors[[i]]}, {i, 1, 4}]]]
countsalt[4] = NIntegrate[F2[x, params[[5, 2]], params[[5 + 5, 2]],
    params[[5 + 10, 2]]], {x, params[[5, 2]] - 2 * fwhm[params[[5 + 5, 2]]],
    params[[5, 2]] + 2 * fwhm[params[[5 + 5, 2]]]}];
countserroralt[4] = Sqrt[paramerrors[[-1]] ^ 2 + peakbackground[[2]] ^ 2];
{countsalt[4], countserroralt[4], 100 * countserroralt[4] / countsalt[4]}
```

{9983.89, 1265.86}

{3.98391×10^{6}, 1519.23, 0.0381341}